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Evaluation of Mathematical Models for the Apparent Mass of the Seated Human Body Exposed to Vertical Vibration

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Systematic Review

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Abstract

In this study, the Authors propose the discussion of nonlinearity of the human body's dynamic response. The variables that affect nonlinearity of the human body's dynamic response in the experimental measurements can be distinguished in two categories: intrinsic variables, relating to the individual subjects; and extrinsic variables, relating to the experimental conditions. International Standard $5982 : 2002$ gives idealized values for the apparent mass and the seat–to–head transmissibility of seated people exposed to vertical vibration. The values are intended for the development of mechanical models to represent the body. Many mathematical models of the vertical apparent mass of the seated human body are developed. Single and two–degree–of–freedom models obtain a good agreement with experimental seat transmissibility by nonlinear least squares method and Trust–Region algorithm. The comparison between single and two–degree–of–freedom models by goodness-of-fit statistics suggests that two–degree–of–freedom model is recommended for best results. [DOI:10.12866/J.PIVAA.2013.12.002] [1]

Keywords: Nonlinear least squares, Statistical analysis, Apparent Mass, Seated Human Body

1 Introduction

The biodynamic response characteristics of seated subjects exposed to vibration have been extensively reported in terms of apparent mass (APMS). The aim in this paper is to propose a comparison of mathematical models for the driving point apparent mass of the seated human body. The

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mechanical impedance of the human body could be represented by a discrete system of masses, springs and dampers or a distributed parameter model. In order to describe the driving point apparent mass over the frequency range of interest, it can be ignored the motions of body parts which do not contribute to the driving point apparent mass. Therefore, the number of degrees of freedom, required in a model, depends on this objective. Seat transmissibilities, obtained with human subjects, can be computed by single–degree–of–freedom or two–degree–of–freedom response in the human body. Involving a re–analysis of the earlier experimental data, this study examines the characteristics of mathematical models. It appears that single–degree–of–freedom or two–degree–of–freedom models could be used to represent the apparent masses of people over the frequency range 0–20 Hz by a non–linear least squares method.

2 Synthesis of the published studies

Simple linear single–degree–of–freedom models are shown in Figure 1. The mass m represents the weight of the person which is supported by tissues represented by the spring k and damping c [Coermann, 1962] and [Wei and Griffin, 1998]. The apparent mass frequency response function is presented in preference to other force response relationships, such as mechanical impedance or dynamic stiffness, because at zero frequency it indicates the static weight of a person on the seat [Wu et al., 1999] (Figure 1).

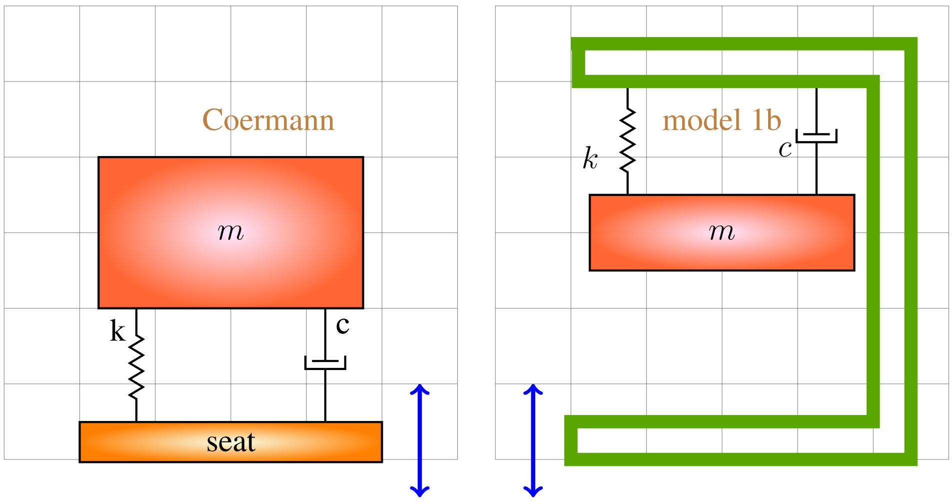

Figure 1: Selected biodynamic representations: model by Coermann and model 1b by Wei and Griffin

In the literature there are many mathematical models for predicting seat transmissibility. Driving–point impedance and the transmissibility from seat to head were analyzed under increased gravity on a centrifuge to determine the nonlinear properties of the upright sitting human body [Mertens, 1978]. Under this condition Mertens observed a shift of resonances to higher frequencies. At the same time the transmissibility increased distinctly. A non–linear multi–degree–of–freedom model of the upright sitting body was proposed to explain this non–linear behaviour.

During the research [Patil and Palanichamy, 1988], the Authors proposed a composite model consisting of a human body, a tractor, and its relaxation seat suspension. The model was subjected to sinusoidal, idealized field or road profile, vibrations at the tyre contact points. The resulting transient and steady state responses of each body part were studied to select the parameters of the relaxation seat suspension.

Even the damping behavior of the seat has been investigated, by using the Bouc–Wen model, Griffin examined all the combined nonlinear effects coming from friction, the damper, different clearances and the seat cushion [Gunstona et al., 2004]. The Bouc–Wen model, having a non–linear degree–of–freedom, is particularly suited to situations where a seat is an input or output to another system.

A generalized nonlinear model is formulated for the dynamic analysis of suspension seats with passive, semi–active and active dampers [Bouazara et al., 2006]. The model non–linearities primarily originate from the damping force, specifically for the semi–active case, the Coulomb friction force, and from the force developed by the elastic stops. The spring rate of the bump stop assumes two different values, in compression and rebound, respectively.

Analyzing the experimental investigations, some subjects have apparent masses showing one degree of freedom while others show two degrees of freedom. For this reason one and two–degree–of–freedom systems were investigated as representations of subject apparent mass.

It is difficult to make a real model like that proposed by Coermann, shown in Figure 1. To prevent rotational modes of vibration, there is no support and no constraint for the mass other than the spring and damper.

An alternative single–degree–of–freedom model (model 1b) is shown in Figure 1. In this model the mass of the person is divided into two parts a support structure m_1 and a sprung mass m_2 [Wei and Griffin, 1998].

As previously mentioned, many experimental investigations have been developed. Griffin provides a comparison between measurements of vibration in a variety of road vehicles and the guidance provided in ISO 2631-1974 (E) (Guide for the evaluation of human exposure to vibration). Some of the problems inherent in comparing the measured vibration levels with the Standard are outlined. Griffin discussed the need for a revised format for the Standard [Griffin, 1978].

In addition to the International Standard ISO 2631, the vibration in vehicles could be measured, evaluated and assessed according to British Standard BS 6841 (1987). In vehicle, Paddan et al. measured the vibration in five axes: vertical vibration beneath the seat, fore–and–aft, lateral and vertical vibration on the seat pan and fore–and–aft vibration at the backrest [Paddan and Griffin, 2002].

Some measurements of the apparent mass of the seated body in the fore–and–aft and lateral directions were already proposed by Fairley and Griffin. The objective was to obtain fundamental data on the horizontal dynamic response of the seated body that could be used to design vibration isolation mechanisms for vehicle seats [Fairley and Griffin, 1990].

To avoid the need for human subjects in seat testing and eliminate the random error caused by inter-subject variability, many Authors proposed the study of mechanical dummies. Mechanical dummies can provide a standard measurement condition. An adaptive dummy, that simulates the driving point response of human subjects over a wide range of different input motion magnitudes, can be produce by an active control.

Firstly, the active vibration dummy simulates the dynamic behaviour of sitting man expressed in terms of the driving point impedance for arbitray body masses and excitation signals. The dummy is realized as a mechatronic system basing on a single degree of freedom setup. A real–time control loop of mass accelerations fit the active dummy to the desired point impedance data set. Model and controller parameters are determined by a parameter–identification technique giving meaningful results for arbitrary impedance data sets. The performance active vibration dummy overcomes limitations of common passive anthropodynamic dummies. The excellent results underline the ability of the established control strategy which eliminates influences of friction forces in the bearing mechanism or any other kind of nonlinear damping and stiffness terms due to the small air gaps in the actuator [Cullman, 2001].

Also, Lewis and Griffin proposed a mechanical dummy with a suitable apparent mass characteristic [Lewis and Griffin, 2002]. Mechanical suspension components, such as dampers, tend to have limitations that modify their dynamic performance when the excitation magnitude is lower, or higher, than an optimum operating range and result in non–linearities in mechanical dummies. Anthropodynamic dummies, based on passive mass–spring–damper systems, have been developed for testing seats but their performance has been limited at low excitation magnitudes by non–linear phenomena, such as friction in the mechanical components that provide damping. To overcome these limitations, Lewis and Griffin propose the use of an electrodynamic actuator to generate damping forces, controlled by feedback from acceleration and force transducers [Lewis and Griffin, 2002].

A further aspect of the research is the low frequency biodynamic behavior of seated humans. Two anthropodynamic manikins were evaluated in the laboratory for their potential applications in assessing vibration isolation effectiveness of seats, and for simulating the low frequency biodynamic behavior of seated humans [Nelisse et al., 2008]. The results of the study generally showed that the vibration isolation effectiveness of a suspension seat is a complex function of the seated mass and nature of excitation (magnitude and frequency contents). Two anthropodynamic manikins are able to investigate the higher resonant frequency and unwanted nonlinearities due to friction or nonlinear damping.

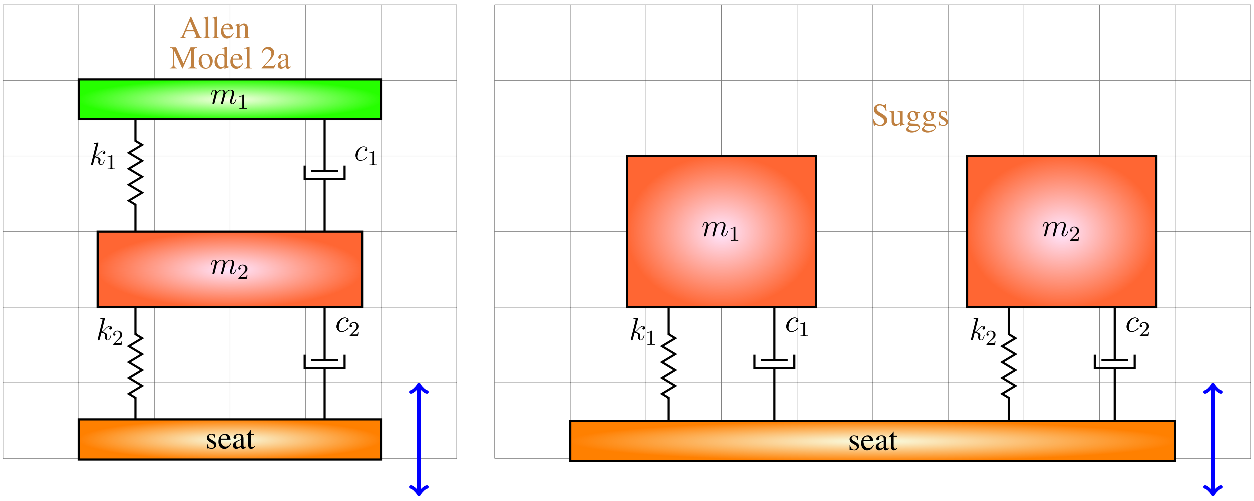

Figure 2: Selected biodynamic representations: model 2a by Wei and Griffin and model by Suggs

With regard to nonlinearities due to friction, progressive endstops is described by the non–linear spring force [Stein et al., 2008]. The purpose of the present study was to demonstrate the effectiveness of the friction–inclusive model in response to the low magnitude motions. The models of the fore–and–aft suspension system is described by a complex model with inclusion of the end–stops.

Since the normalized APMS function tends to suppress the magnitude of APMS at higher frequencies, the magnitude corresponding to the second resonance frequency of higher order models, in general, tends to be considerably lower. This observation justifies the two–d.o.f. model, proposed by [Allen, 1978] (Figure 2).

If a dummy were manufactured according to this model, a constraint mechanism would be required to ensure that the sprung mass m_2 moved only in the vertical direction. A two–degree–of–

freedom system having a support structure is shown in Figure 3 (model 2b). It has two mass–spring systems m_1 and m_2 supported on the support mass m. It is tempting to assume that the mass m_2 consists of the masses of the head and the upper torso while the mass m_1 represents the main part of the body and the mass m comprises the skeleton.

The mathematical models represents equivalent mechanical systems with a mechanical impedance similar to that of the human body. However, the models are not intended to represent the locations or mechanisms of body movement. Measurements of the mechanical impedance of the human body usually show evidence of a two–degree–of–freedom response. For this reason it seems appropriate to consider a two–degree–of–freedom system.

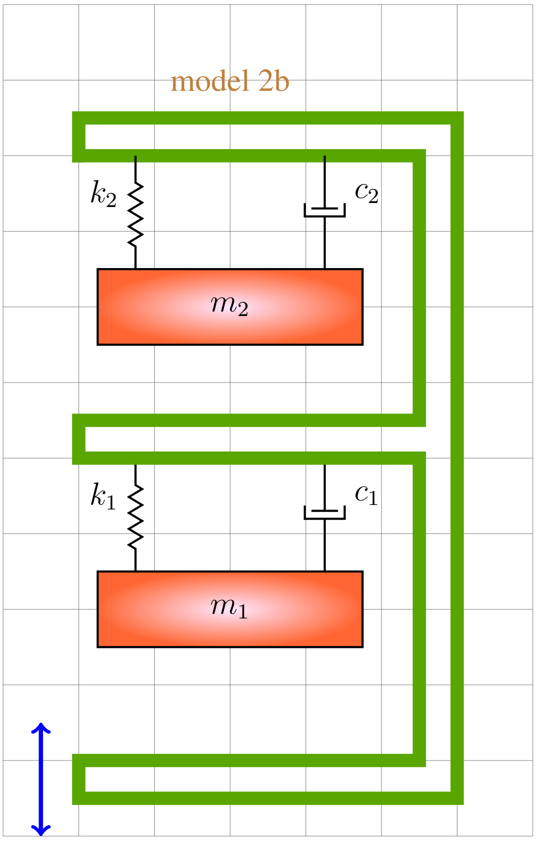

Figure 3: Selected biodynamic representation: model 2b by Wei and Griffin

In the literature are proposed more comprehensive mathematical models. The development of complex models of the responses of the body requires an understanding of the modes of oscillation of the body. A non–dimensional mechanical–equivalent model is proposed to characterize both the APMSs, and its validity is demonstrated through comparisons of the seat pan and backrest APMS responses with the mean measured data for occupants within different body mass ranges [Rakheja et al., 2006].

A lumped parameter model with a vertical, a horizontal and a rotational degree–of–freedom has been developed to fit the moduli and phases of the vertical apparent masses and fore–and–aft–cross–axis apparent masses of persons sitting on a rigid seat with no backrest during vertical excitation [Nawayseh and Griffin, 2009].

A three lumped–parameter models from literature have also been analyzed and optimized using genetic algorithms to match an experimental data in terms of STH transmissibility, DPM impedance, and AP mass [Abbas et al., 2010].

A body–seat model has been developed to mach both the fore–and–aft apparent mass at the back and the backrest transmissibility [Qiu and Griffin, 2011].

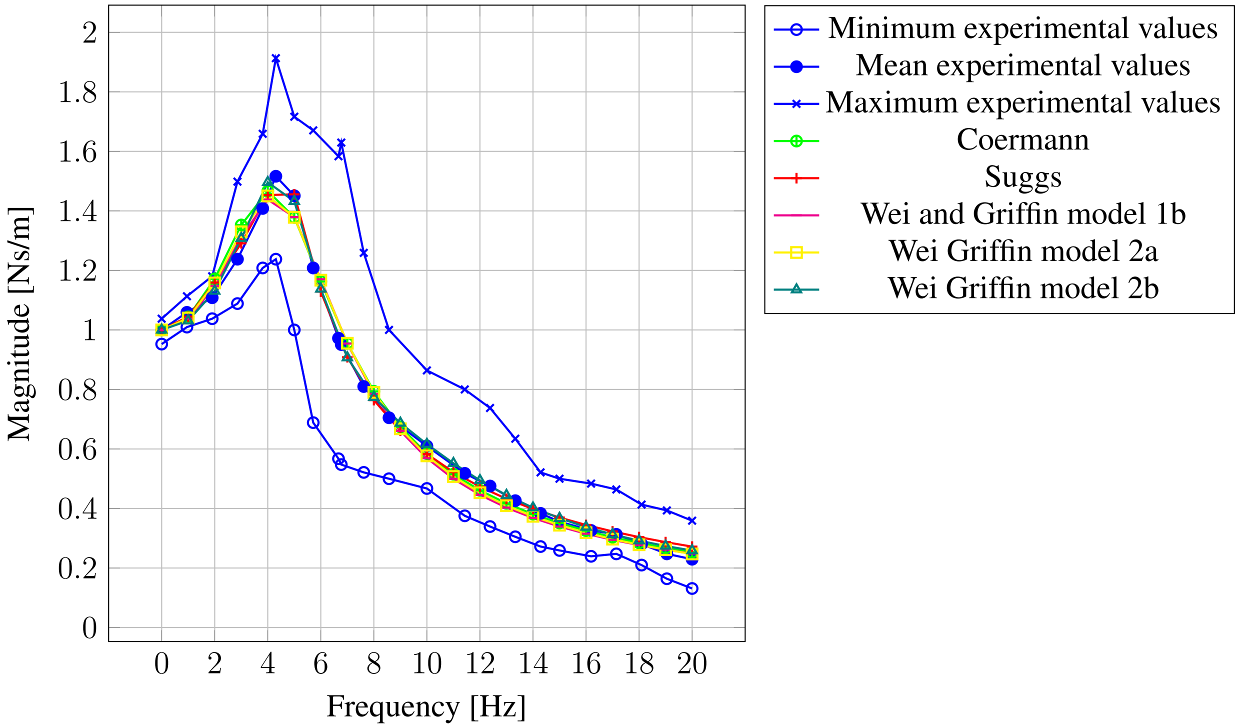

Figure 4: A comparison of the normalized APMS magnitude measured and computed in the vertical direction

3 Statistical analysis

The vertical whole–body driving–point apparent masses of 60 persons were obtained with the subjects seated on a rigid force platform without a backrest. Subjects were exposed to 1.0 ms^{-2} random vertical vibration over the range 0–20 Hz. The subjects sat in a normal upright posture with their feet supported on a footrest which vibrated in phase with the seat.

The main purpose of the present study is to compare mathematical models of the apparent mass of the seated human body for use in procedures for predicting seat transmissibility. Fitting data with nonlinear least squares method and Trust–Region algorithm, the goodness of fit is evaluated by following parametric models: the sum of squares due to error (SSE), R–square, adjusted R–square and root mean squared error. Goodness-of-fit statistics proposes numerical measures that aid statistical reasoning. The numerical measures, obtained by parametric models, are more narrowly focused on a particular aspect of the data. the peculiarity i that the numerical measures condense information into a single number. The definition of parametric models are illustrated in Table 2.

By nonlinear least squares method we compare single and two–degree–of–freedom mathematical models of different Authors. As single–degree–of–freedom mathematical models, we select two types of biodynamic models:

- *Single–degree–of–freedom mathematical models*. In the Figure 1 the first one is proposed by

Coermann [Coermann, 1962] and second one is propose by Wei and Griffin [Wei and Griffin, 1998].

- *Two–degree–of–freedom mathematical models*. In the Figure 2 it is represented the models recommended by Suggs [Suggs et al., 1969], by Allen [Allen, 1978] and the model 2a recommended by Wei and Griffin [Wei and Griffin, 1998]. In the Figure 3 it is represented the model 2b suggested by Wei and Griffin [Wei and Griffin, 1998].

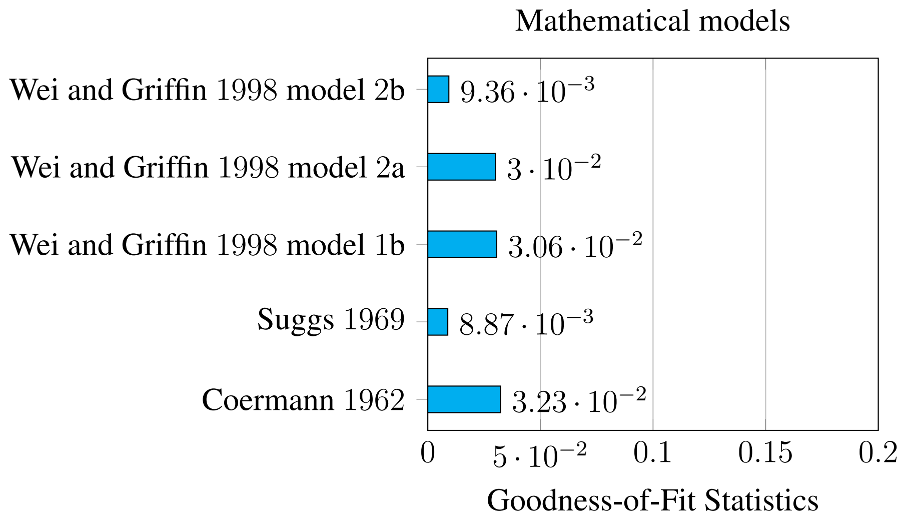

Figure 5: The comparison of the sum of squares due to error (SSE)

The equations of motion, formulated for the five models, shown in Table 1, are further analyzed to derived expressions for the normalized APMS magnitude function. Table 1 summarizes the expressions derived for normalized APMS magnitude and phase functions for the five different models considered in the study. The normalization of APMS was realized upon dividing the APMS magnitude by the total mass of the model. Although the models do not describe a direct biomechanical representation of the head, the response of the mass at the extremity is taken to describe the motion of the head (mass m in the single-d.o.f. models, and mass m_1 in the two-d.o.f. models).

Wei and Griffin proposed the apparent masses of 60 seated subjects and derived a single and two–degree–of–freedom model to fit the measured data [Wei and Griffin, 1988]. By nonlinear least squares method we obtain a good agreement between normalized APMS magnitude and seat transmissibilities, obtained with human subjects (Fig.4). With reference to the results of goodness-of-fit statistics (Figures 5, 6, 7 and 8), the single–degree–of–freedom model and the two–degree–of–freedom model both provided results close to the measured modulus of apparent mass.

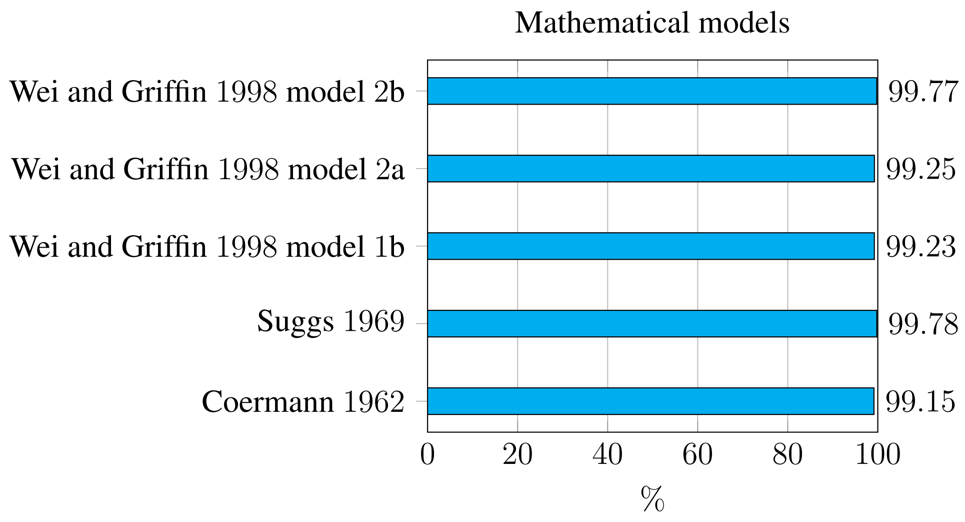

Figure 6: The comparison of the R–square

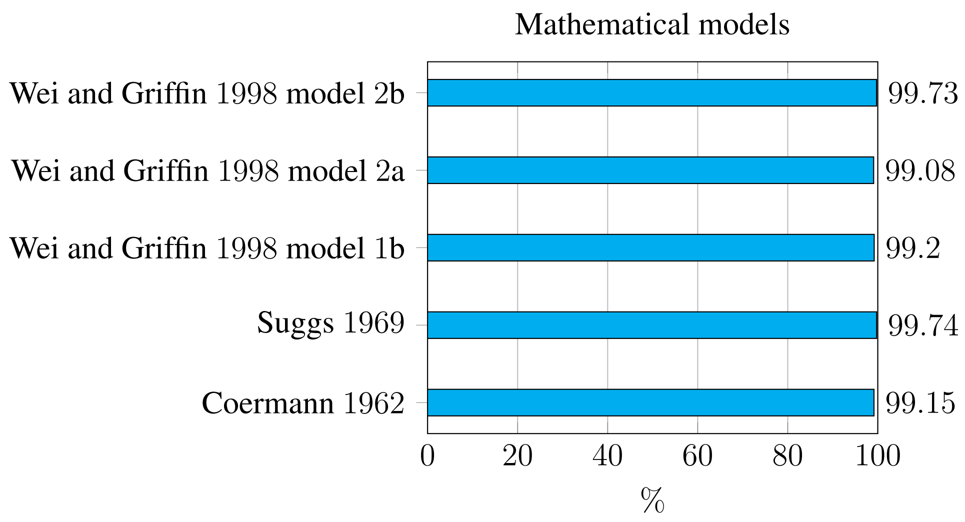

Figure 7: Fig:The comparison of adjusted R–square

If all mathematical models predict successfully seat transmissibility, experimental seat transmissibility often show evidence of two–degree–of–freedom response in the human body. Two–degree–of–freedom mechanical model provides a better fit to the measured data than a single–degree–of–freedom model. Application of the two–degree–of–freedoom model provides a better fit to the magnitude data in the frequency range 0-20 Hz. Therefore, two–degree–of–freedom model is recommended for best results. To predict the transmissibility of seats it is recommended the two–degree–of–freedom model with a support mechanism (model 2b).

4 Discussion

An interesting study measured the dynamic responses of seated people at locations to define the form of body movements during exposure to vertical whole–body vibration [Matsumoto and Griffin, 1998].

Large differences between subjects are particularly apparent in the vertical seat–to–head trans-

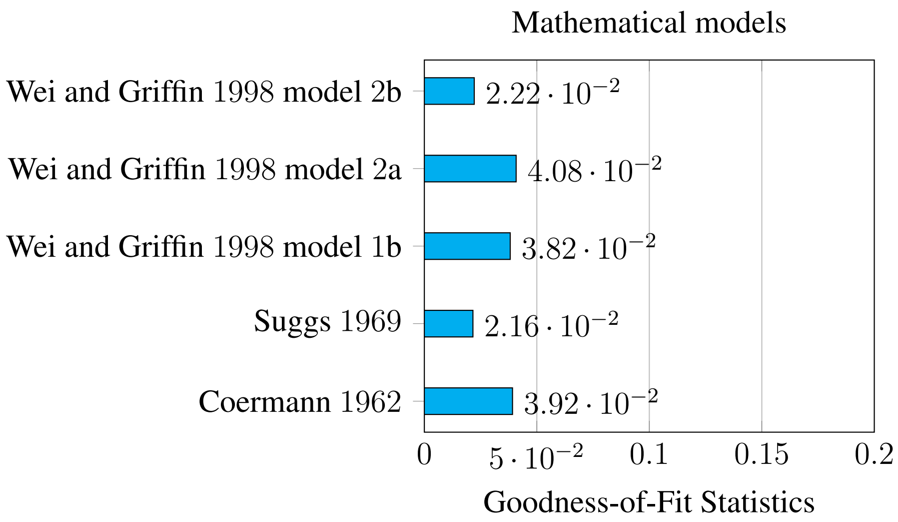

Figure 8: The comparison of root mean squared error (RMSE)

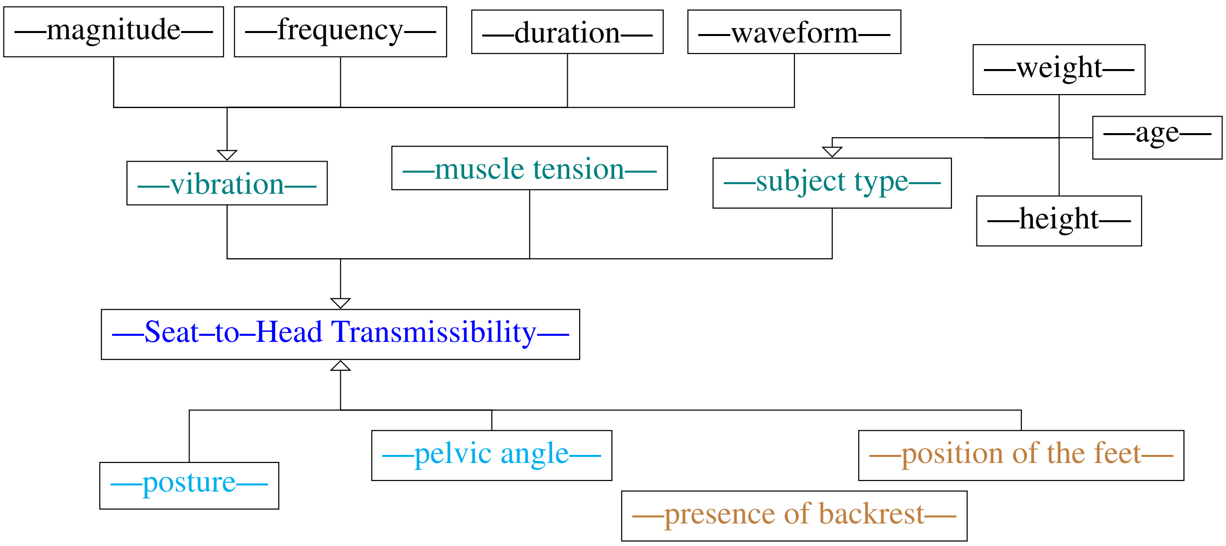

Figure 9: Intrinsic and extrinsic variables relating to the individual subjects

missibility curves shown in Fig. 4. At 6.5 Hz, one subject had a z–axis seat transmissibility differs from 1.0 to 1.6. One possible reason for the scatter is the different static characteristics of the subjects. The transmissibility depends on contact with the back of the seat which represents the additional transmission path for vibration [Paddan and Griffin, 1988].

The variables (Table 9) that affect the experimental measurements can be distinguished in two categories:

- intrinsic variables, relating to the individual subjects;
- and extrinsic variables, relating to the experimental conditions.

Stiffness and damping coefficients k_1, k_2, c_1 and c_2 vary as a function of age, total mass and of the male and female gender, and children (Figures 10, 12, 11, and 13).

Some variables can have large effects on seat–to–head transmissibility: sitting posture (pelvic angle, head angle), contact with the seat backrest, variations in posture, muscle tension, subject type (age, weight and height) [Paddan and Griffin, 1998].

The position of the feet can adapt the degree to which the thighs were in contact with the seat and also the pressure on tissues beneath the pelvis. The mass, supported on the footrest, can increase from the maximum thigh contact posture to the minimum thigh contact posture. Contracting the area of contact at the seat, the mass, supported on the seat, can diminish, increasing the pressure on the pelvis tissue and the stiffness of these tissues [Nawayseh and Griffin, 2003].

Most studies have concluded that the APMS response depends upon the vibration magnitude, and a *softening* of the body with increasing excitation magnitude, while the effect is nonlinear. Therefore, the biodynamic responses of the seated body to whole-body vibration in terms of apparent mass have been shown to exhibit nonlinearity with respect to vibration magnitude [Wang et al., 2008].

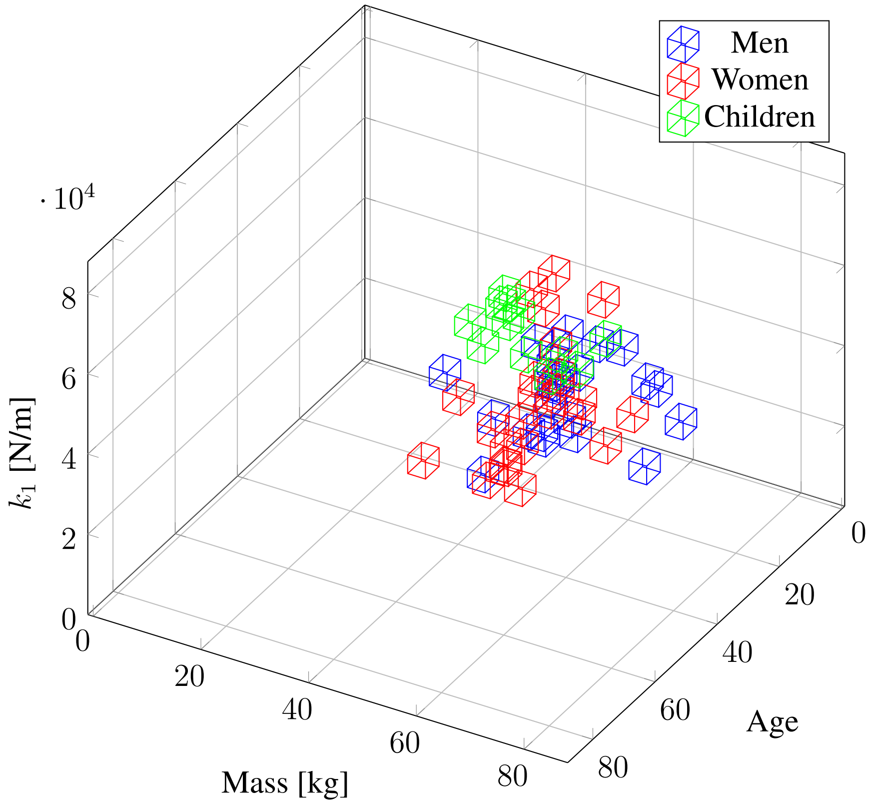

Figure 10: Stiffness coefficient k_1 in function of age and of total mass of subject

It has thus been suggested that the biodynamic models should be developed to reflect such non–linear dependency. The effect of magnitude of excitation on the biodynamic responses of occupants seated assuming a typical automotive posture was observed to be slightly more important for the passenger posture (hands in lap) than for the driving posture (hands on steering wheel) (Rakheja et al., 2002).

Experimental results shows that hands in lap postures are relatively more sensitive to vibration excitation magnitude. The APMS magnitudes in general tend to decrease with an increase in excitation magnitude. The body *softening* effect tends to be more apparent under no back support posture, irrespective of the hands position. The results indicate that the primary resonance reduces by 0.5–0.6 Hz with increasing excitation magnitude from 0.5 to 1.0 m/s^2 rms, which is found to be significant. The corresponding reductions in primary resonance under vertical back and inclined back support postures were found to be in the order of 0.3 Hz and less than 0.2 Hz [Wang and Boileau, 2004].

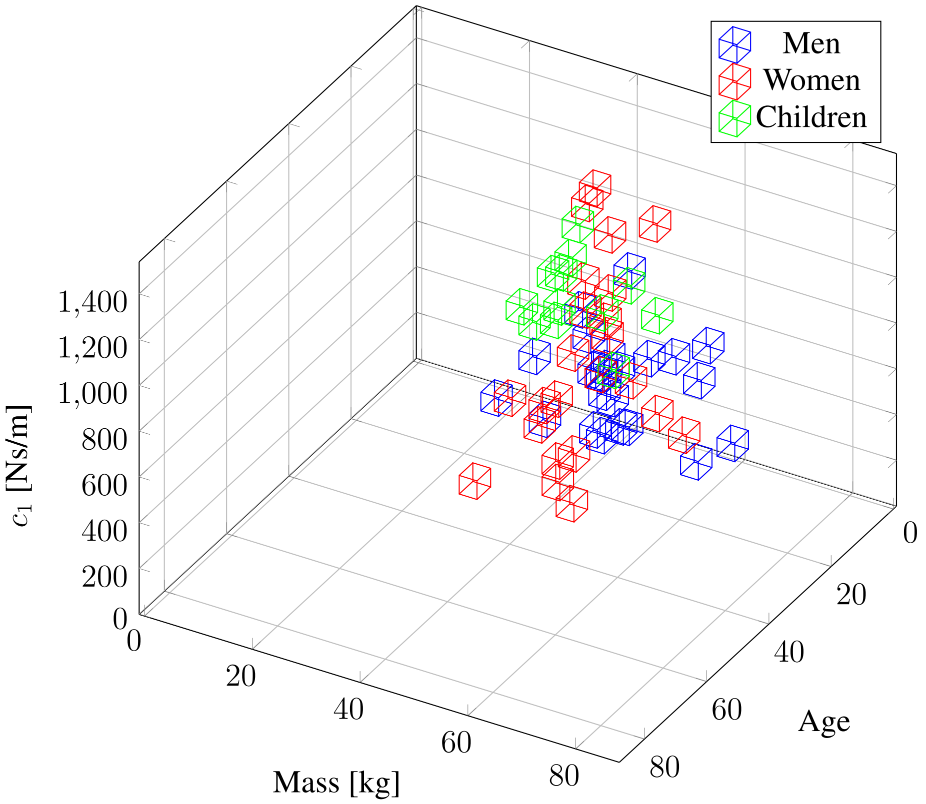

Figure 11: Stiffness coefficient c_1 in function of age and of total mass of subject

The results suggest that the back support condition strongly affects both the seat–to–head transmissibility and apparent mass responses. Furthermore, the hands position may also influence both the responses, while little has been reported in the literature on the influences of such factors. The human body shows a highly nonlinear response to vibration. Mansfield and Griffin demonstrated the nonlinearity of the seat vibration transmitted to various body segments of the seated body [Mansfield and Griffin, 2002], namely the viscera, pelvis and lumbar spine [Wang et al., 2008].

The frequency of the peak is a function of the vibration magnitude. This non–linearity is characterised by the resonance frequency decreasing as the vibration magnitude increases. Mansfield et al. have shown that the biomechanical response of the seated human is non–linear. The waveform of vibration is an important consideration when measuring the apparent mass, especially considering the nonlinear response of the body and the difficulty in comparing magnitudes of vibration when presented as either sinusoidal or random [Mansfield and Maeda, 2005].

The question of nonlinearity of the human body's dynamic response has remained open:

- If the measured response amplitude is not directly proportional to the amplitude of the driving motion, while the frequency is held constant, it can be stated that the system is nonlinear. This aspect of the nonlinear response would speak in favour of a system with a variable coupling, and its elastic and damping properties would depend on the input parameters. Mechanical properties of the soft tissue, relative motions of body parts, and muscle reactions were supposed to cause the nonlinearities of the head [Hinz et al., 2010].

- If a sinusoidal input acceleration does not produces a sinusoidal output measured as transmitted force or acceleration, it can be stated that the system is nonlinear. It can examine an another characteristic for the nonlinearity. In fact, a characteristic for the nonlinearity is a trasformation of the sinusoidal mechanical input (external stress) into a nonsinusoidal dynamic response possibly linked with high peak values of the latter and muscular reactions [Hinz and Seidel, 1987].

The nonlinearity in the vertical apparent mass is consistent with the decrease in the resonance frequency, with an increase in vibration found previously in measures of apparent mass and mechanical impedance and in measures of transmissibility to different parts of the body when horizontal supporting seat surfaces were used. Matsumoto and Griffin noticed a decrease in the nonlinearity when subjects sat with tensed buttocks tissue compared to a normal sitting posture, which implies that these tissue are partly responsible for the nonlinearity [Matsumoto and Griffin, 2002]. Nawayseh and Griffin reached the same conclusion when they noticed a decrease in the nonlinearity when the upper body was resting on the tissue of the buttocks in a minimum thigh contact posture compared to other postures with greater thigh contact [Nawayseh and Griffin, 2005]. It was expected that increasing the seat surface angle would increase the shear stiffness of the ischial tuberosities and this might affect the nonlinearity in the vertical direction [Nawayseh and Griffin, 2005].

The geometry of the backrest modified the forces in vertical direction on the supporting seat surface. This study reveals that contact with a backrest, and the inclination and compliance of a backrest, modify the vertical apparent mass of the human body measure data seat surface [Toward and Griffin, 2009, 2010].

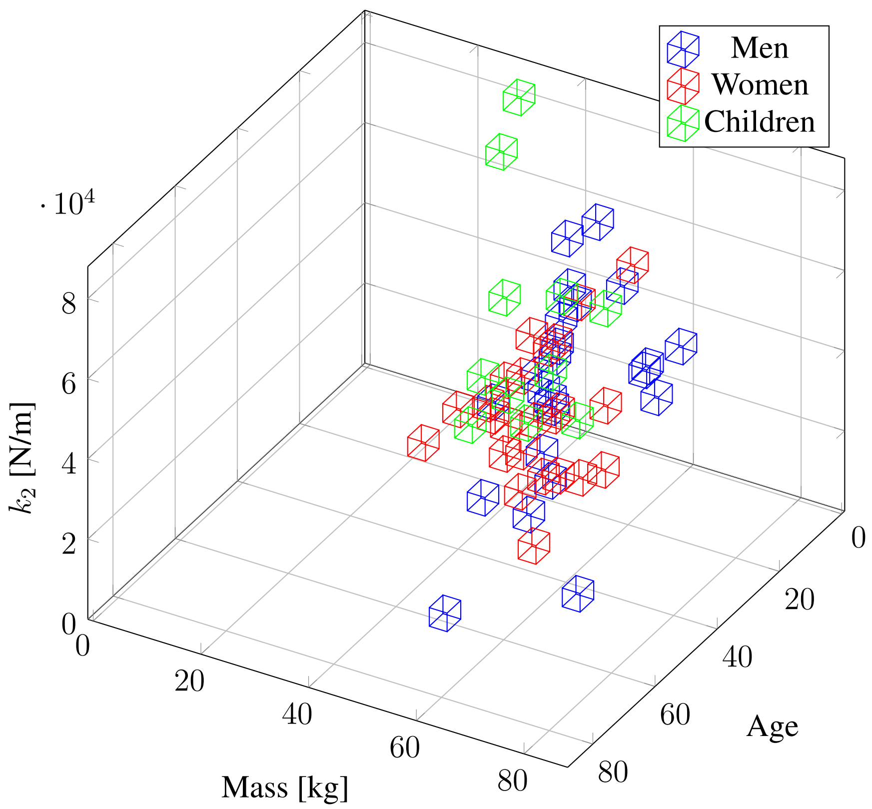

Figure 12: Stiffness coefficient k_2 in function of age and of total mass of subject

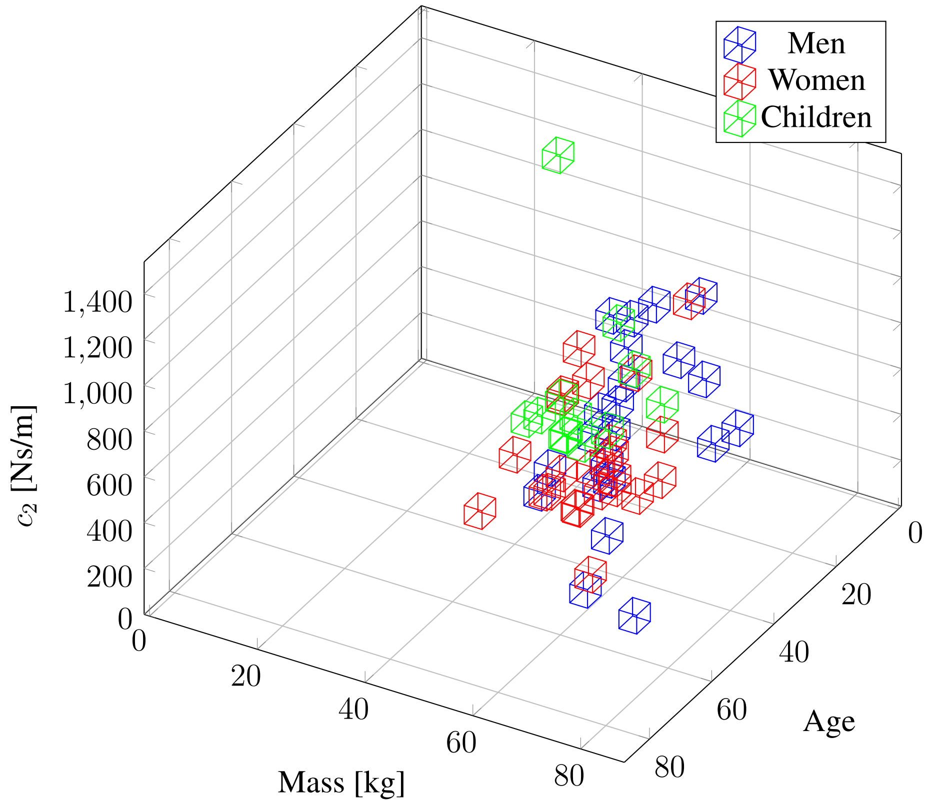

Figure 13: Stiffness coefficient c_2 in function of age and of total mass of subject

5 Conclusions

The interface seat - driver significantly influence the body's response to stress. The seat helps to dampen the dynamic response of the system before the external stress may disturb the person.

In the vertical axis all seats possessed a low frequency resonance which amplified the vibration at some frequencies below 10 Hz. Seat transfer functions suggest that the resonant frequency is usually close to 4 Hz. Several of the transmissibilities appear to reflect two resonances and suggest that the man–seat system is behaving like a two–degree of freedom system with one mass spring system acting as a vibration absorber.

Driving of vehicles involves not only exposure to harmful WBV but also to several ergonomic risk factors which can affect the spinal system, such as prolonged static posture. Individual characteristics (age, anthropometric data, constitutional susceptibility), psychosocial factors, and previous back traumas are also recognized as potential predictors for low back pain. It follows that injuries in the lower back of professional drivers may be considered a complex of health disorders of multifactorial origin involving both occupational and non–occupational stressors.

The Italian Legislative Decree no.$187/19 - 2005$ August implements the directives on health and safety of workers exposed to mechanical vibration. The item 3 of Legislative Decree defines the daily exposure limit value and the value in action daily.

International Standard $5982 : 2002$ gives idealized values for the apparent mass and the seat-to-head transmissibility of seated people exposed to vertical vibration. The values are intended for the development of mechanical models to represent the body [ISO]. The target is an amalgamation of several datasets obtained in broadly comparable conditions.

In the literature there are many mathematical models. It would be possible to develop mathematical models of the driving point impedance of the body having more than two degrees–of–freedom but the results shown here suggest that this is unnecessary when representing the average response of a group of subjects to a specific vibration input. A greater number of degrees of freedom may be required to explain the movements of the body responsible for apparent mass or predict the transmission of vibration through the body.

Curve fitting has allowed the development of mathematical models which provide a good fit to measured values of the normalized apparent masses of subjects.

However, seat transmissibilities obtained with human subjects often show evidence of a two–degree–of–freedom response in the human body. This study involved a reanalysis of the earlier data so as to obtain an improved fit to the measured apparent masses of subjects. To predict the transmissibility of seats it is recommended the two–degree–of–freedom mathematical model with a support mechanism (model 2b).

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Investigator	Normalized APMS
Model	Normalized $M(\omega) = \sqrt{\dfrac{[A(\omega)]^2 + [B(\omega)]^2}{[C(\omega)]^2 + [D(\omega)]^2}}$ $\theta = \arctan \dfrac{B(\omega)}{A(\omega)} - \arctan \dfrac{D(\omega)}{C(\omega)}$
[Coermann, 1962]	$A(\omega) = k, \quad B(\omega) = D(\omega) = c\omega, \quad C(\omega) = (k - m\omega^2)$
[Wei and Griffin, 1998] model 1b	$A(\omega) = k_1 - \dfrac{m_1 m_2}{(m_1 + m_2)}\omega^2$ $B(\omega) = \omega c_1$ $C(\omega) = (k_1 - m_2\omega^2)$ $D(\omega) = c_1\omega$
[Suggs et al., 1969]	$A(\omega) = k_1 - \left[\dfrac{c_1c_2}{k_2} + \dfrac{(m+m_1)\,m_2}{m+m_1+m_2}\dfrac{k_1}{k_2} + \dfrac{(m+m_2)\,m_1}{m+m_1+m_2}\right]\omega^2$ $+ \dfrac{mm_1m_2}{m+m_1+m_2}\dfrac{1}{k_2}\omega^4$ $B(\omega) = \left(c_1 + c_2\dfrac{k_1}{k_2}\right)\omega - \left[\dfrac{(m+m_1)\,m_2}{m+m_1+m_2}\dfrac{c_1}{k_2} + \dfrac{(m+m_2)\,m_1}{m+m_1+m_2}\dfrac{c_2}{k_2}\right]\omega^3$ $C(\omega) = k_1 - \left(m_1 + m_2\dfrac{k_1}{k_2} + \dfrac{c_1c_2}{k_2}\right)\omega^2 + \dfrac{m_1m_2}{k_2}\omega^4$ $D(\omega) = \left(c_1 + c_2\dfrac{k_1}{k_2}\right)\omega - \left(\dfrac{m_1c_2 + m_2c_1}{k_2}\right)\omega^3$
[Wei and Griffin, 1998] model 2a [Allen, 1978]	$A(\omega) = k_1k_2 - \left(c_1c_2 + \dfrac{m_1m_2}{m_1+m_2}k_1\right)\omega^2$ $B(\omega) = (c_1k_2 + c_2k_1)\,\omega - \dfrac{m_1m_2}{m_1+m_2}c_1\omega^3$ $C(\omega) = k_1k_2 - (m_1k_2 + m_2k_1 + m_2k_2 + c_1c_2)\,\omega^2 + m_1m_2\omega^4$ $D(\omega) = (c_1k_2 + c_2k_1)\,\omega - (m_1c_2 + m_2c_1 + m_2c_2)\,\omega^3$
[Wei and Griffin, 1998] model 2b	$A(\omega) = k_1k_2 - \dfrac{(mm_2k_1 + mm_1k_2 + m_1m_2k_1 + m_1m_2k_2)}{(m+m_1+m_2)}\omega^2$ $+\dfrac{mm_1m_2}{(m+m_1+m_2)}\omega^4 - \dfrac{(mc_1c_2 + m_1c_1c_2 + m_2c_1c_2)}{(m+m_1+m_2)}\omega^2$ $B(\omega) = (k_1c_2 + k_2c_1)\,\omega - \dfrac{(mm_1c_2 + mm_2c_1 + m_1m_2c_2 + m_1m_2c_1)}{(m+m_1+m_2)}\omega^3$ $C(\omega) = k_1k_2 - \omega^2(k_1m_2 + k_2m_1) + m_1m_2\omega^4 - c_1c_2\omega^2$ $D(\omega) = (k_1c_2 + k_2c_1)\,\omega - (m_1c_2 + m_2c_1)\,\omega^3$

Table 1: Expressions for the magnitude and phase of the normalized APMS of the selected models

Symbol	Definition	Variation range
R–square	R–square is the square of the correlation between the response values and the predicted response values. R–square is defined as the ratio of the sum of squares of the regression (SSR) and the total sum of squares (SST). SSR is defined as $SSR = \sum_{i=1}^{n} w_i \left(\hat{y}_i - \bar{y}\right)^2$. SST is also called the sum of squares about the mean, and is defined as $SST = \sum_{i=1}^{n} w_i \left(y_i - \bar{y}\right)^2$, where SST = SSR + SSE. Given these definitions, R–square is expressed as R–square $= \frac{SSR}{SST} = 1 - \frac{SSE}{SST}$.	R–square can take on any value between 0 and 1. A value closer to 1 indicating that a greater proportion of variance is accounted for by the model.
Sum squared error (SSE)	This statistic measures the total deviation of the response values from the fit to the response values, defined as $SSE = \sum_{i=1}^{n} w_i \left(y_i - \hat{y}_i\right)^2$.	A value closer to 0 indicates that the model has a smaller random error component, and that the fit will be more useful for prediction.
Adjusted R–square	The adjusted R–square statistic is generally the best indicator of the fit quality. By this statistic we compare the mathematical models respect to mean experimental values. It is defined as adjusted R-square $= 1 - \frac{SSE\,(n-1)}{SST\,(v)}$.	The adjusted R–square statistic can take on any value less than or equal to 1, with a value closer to 1 indicating a better fit.
Root mean squared error (RMSE)	This statistic can be defined as the fit standard error and the standard error of the regression. It is an estimate of the standard deviation of the random component in the data, and is defined as $RMSE = s = \sqrt{MSE}$, where MSE is the mean square error or the residual mean square $MSE = SSE/v$.	Just as with SSE, an MSE value closer to 0 indicates a fit that is more useful for prediction.

Table 2: Goodness-of-Fit Statistics