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Evaluation of psycometric function of vertical whole–body vibration on a rigid seat by statistical distribution functions

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Systematic Review

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Abstract

The psychometric function relates an observer's performance to an independent variable, usually some physical quantity of a stimulus in a psychophysical task. We consider the peception of whole–body vibrations just with vertical sinusoidal excitations. This paper describes an integrated approach to fitting psychometric functions and assessing the goodness of fit. [DOI:10.12866/J.PIVAA.2015.06.01] [1]

Keywords: Psychometric function, Discomfort, Statistical procedures

1 Introduction

The psychometric function performs a basic role in the context of the vibrations on the human body. The human body is exposed to many sources of vibration in all types of transport, in buildings, and from the operation of industrial equipment. People react to the vibration according to their perception. The psychometric function describes the relationship between the level of a stimulus and a subject's response [Gescheider et al., 2009]. The response represents the probability of success at that stimulus level on a certain number of trials. The success is evaluated on two possible forms: present or absent stimulus, longer or shorter stimulus.

The psychometric function has a peculiar aspect. The magnitude of the vibration to which the body is exposed can be expressed in terms of physical measurements (displacement, velocity, or

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acceleration). Therefore, the vibration is a quantitative measurement of a physical quantity. The physical magnitude of the motion may be expressed in meters per second per second. For against, the sensations, experienced by people, must be obtained using psychophysical measures: rating of perceptibility, comfort, annoyance, or pain. Psychophysical measurements have nominal, ordinal, interval, or ratio characteristics. The psychometric function can be described by a specific parametric model. The values of parametric model are optimized by using the maximum–likelihood estimation.

Important questions regard the existence, the nature, the measurement and the representation of the psychometric function [Aghilone and Cavacece, 2015]. The practical purposes are to reduce vibrations in order to be perceived less uncomfortable. The knowledge of sensory threshold allows vibration reduction. The second question is the nature of the psychometric function that permits a concise description of empirical data. The description of the psychometric function offers predictions about sensory performance. In order to obtain the probabilistic description of the psychometric function, experimental values are represented by using a continuum function. The question of how the psychometric function can be determined has acquired interest through the development of adaptive threshold measurement techniques by using maximum likelihood or Bayesian principles. These methods depend on the assumptions about the psychometric functions shape, expressed by its parameters. These assumptions have to be checked.

This paper considers three major subsections: psychometric functions, experimental procedures and goodness of fit. The psychometric function describes the percentage of perceived stimuli that can be delineated by logistic distribution or by Weibull distribution. The statistic approach to the psychometric functions is integrated by evaluating the goodness of fit.

2 Statistical approaches and procedures

The method of constant stimuli is useful to evaluate the absolute threshold by a statistical distribution function. The experimental investigations, proposed in the scientific literature, illustrated several procedures. It is helpful to have a measure of goodness of fit to assess the adequacy of the estimated psychometric function in accounting for the data.

2.1 Method of constant stimuli

As regards the sequence of stimuli, the method of constant stimuli proposes various stimulus levels to the subject in random or quasi–random order. This method is a nonsequential procedure. The stimuli are not presented in ascending or in descending order. In order to obtain an optimal estimate of the threshold, each stimulus should be proposed, in random series, many times. The number of stimuli, proposed to the observer, are equal to the number of discrete levels multiplied by the number of repetitions of each level.

With reference to the task, the subjects have to compare two different stimuli with different frequencies and different levels. The subjects indicate whether the stimulus presentation has been perceived during each trial. On each trial, the observer is asked to report whether or not he acquired a stimulus. After each stimulus presentation, the observer indicates the following aspects:

- whether or not the stimulus was detected to define the absolute threshold;
- whether its intensity was stronger or weaker than standard stimulus for computing a difference threshold.

The proportion of detected and not detected (or, stronger and weaker) stimulus, acquired for each stimulus level, is evaluated. The assessment, calculated for each tone intensity, is estimated by the probability of detection, or percentage of *yes* responses.

The psychometric function is plotted with stimulus intensity along the abscissa and percentage of perceived stimuli along the ordinate. It obtains a sigmoid curve. Lower stimulus intensities are detected occasionally. Higher values are detected more often. A major source of variability of psychometric function are the continual fluctuations in sensitivity and the internal noise, present in any biological sensory system. The threshold occurs with a certain probability and its intensity value must be defined statistically. By convention, the absolute threshold, measured with the method of constant stimuli, is defined as the intensity value that elicits perceived responses on 50% of the trials.

Particular aspects characterize the method of constant stimuli:

- i–intervals are presented in one trial;
- for each trial only one interval includes a stimulus with varying level;
- for each trial $(i-1)$–intervals comprise no signal.

The task of the subjects is to say if they feel a vibration. The subjects indicate that interval, in which they felt the vibration.

With reference to the favorable and the unfavorable factors, the advantages of the method of constant stimuli are the following factors:

- The precision of the measurement.
- Each stimulus level offers observations.
- A preliminary experiment establishes the upper and lower limits of the range of investigation. The advantage of this procedure is that examine a wide range.
- The stimuli are produced in random order so that the subject cannot realize in advance the stimulus sequence. The order of the observations is randomized.

The method of constant stimuli has some disadvantages:

- A very large number of trials are needed to obtain results.
- Several stimulus levels are proposed by the experimenter. Major drawback of the method of constant stimuli is time–consuming. It needs a large amount of time to execute experimental trials. The method requires a patient, attentive observer because of the many trials required.
- The method is inefficient to estimate only one point on the curve.
- The method involves a large proportion of the observation, placed at some distance from the region of interest [Levitt, 1971].
- The psychometric functions plot the signal strength on the horizontal axis and the probability of the observer saying *yes* on the vertical axis. The fifty percent point is commonly used as an estimate of threshold. The psychometric functions increase gradually with tone intensity. All of the trials are signal trials. There are no catch trials (blanks, noise–alone trials). It can make no estimate of false alarms. We need to know both the hit rate and the false alarm rate to determine detectability. As a result, we cannot use the data from a yes–no experiment to estimate the sensitivity separately from the observer's criterion.

2.2 Statistical distribution function

The psychometric function indicates the subjective response behavior of an individual or group of individuals, depending on given stimulus parameter in a psychophysical experiment. The correct response to detect a stimulus, function of the level x, is represented by the probability $p(x)$. The probability $p(x)$ of detected stimulus is a result, obtained from development of correct responses at a fixed stimulus level x. The psychometric function represents the probability of correct responses for different stimulus levels or magnitude. Stimuli with different levels are presented to a subject with some stimulus repetitions. The psychometric function is obtained by using the subjects' responses. The interpolated function of the measured data defines the psychometric function.

Psychometric function seems an *ogive*. The psychometric function has asymptotes zero at $p(x) = 0$ and one at $p(x) = 1$. In statistics, an ogive is a curve of a cumulative distribution function. The logistic and the Weibull distributions are the most common cumulative distribution functions [Treutwein and Strasburger, 1999]. In our simulations and reanalysis of psychophysical data, we propose the comparison between the logistic distribution and Weibull distribution.

The percentage of perceived stimuli can be described by logistic distribution or Weibull distribution f. The distribution function f can be denoted by the following equation:

$$f_{\vartheta_1,\vartheta_2,\ldots,\vartheta_n}(x_1, x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n|\vartheta) \ . \tag{1}$$

where ϑ_1, ϑ_2, ..., ϑ_n are optimal parameters that maximizes the probability of likelihood between the observed data and the frequency probability distribution f along the abscissa x_1, x_2, ... and x_n.

Many measurements can define the whole shape of a psychometric function at many different levels. A second possibility is to measure some points, especially around the expected absolute threshold. Some of the presented stimuli, with varied level, are detectable for the subjects and some are not detectable. This measurement can be understood as a Bernoulli process with a correct response–probability $p(x)$.

The Bernoulli process is a discrete process. The Bernoulli process consists of a succession of tests with equiprobable and independent events with probability $p(x)$.

The fit of the whole shape can be obtained by using a logistic function. As mentioned before, the whole shape of the psychometric function can be fitted with just some measured points by using a logistic function. The maximum likelihood estimation is used to fit the psychometric function to the measured data.

- *Logistic function.* The Logistic distribution with location a and scale b has density

$$f(x, a, b) = \left(\frac{1}{b}\right) \cdot e^{\dfrac{x-a}{b}} \cdot \left[1 + e^{\left(\dfrac{x-a}{b}\right)}\right]^{-2} \ . \tag{2}$$

  Logistic functions is represented by using an unnormalized abscissa x. The level x_{50} indicates the highest slope and is the central point of the logistic function. That means that x_{50} is the point with 50% correct response–probability $p(x) = 50\%$. The probability $p(x) = 100\%$ indicates that all presented stimuli are detected.

- *General model Weibull.* The Weibull function for f gives the probability p of choosing an individual value at random. The method may be applied to the large group of problems [Weibull, 1951]. The general conditions are the following ones:

- positive function;
- nondecreasing function;
- the order of the observations can be randomized.

In addition, the Weibull function provides a good model for contrast discrimination and detection data [Weibull, 1951]. The Weibull distribution with shape parameter c and scale parameter d has density given by

$$f\left(x,c,d\right)=\left(\frac{c}{d}\right)\cdot\left(\frac{x}{d}\right)^{c-1}\cdot e^{\left(\frac{x}{d}\right)^{c}} \quad \text{with} \quad x>0\,. \tag{3}$$

2.3 Experimental procedures

In the scientific literature we have identified several ways of the experimental procedures (Table 1). We summarize the peculiar aspects:

✓ In the investigation of Michael A. Bellmann, the psychometric function is measured with an interval of $f = 5$ Hz. The observed population is the group consisting of 14 subjects (2 female and 12 male) (Fig.1). The measuring method is a constant stimulus method with three given intervals for each trial. Two of the three intervals include no signal. One interval applies a stimulus with varying level and with a fixed test–frequency of $f = 5$ Hz. The task of the subjects is to mark that interval, in which they felt a vibration. The presented levels L_{vib} vary from 75 to 90 dB, below and above the expected perception threshold. The step is 1.5 dB. A step–size of 1.5 dB is used because a study of Morioka and Griffin reports that the just noticeable difference in level for 5 Hz is about $1/1.5$ dB. Therefore each subject individuates all presented stimuli above the individual threshold. The stimulus duration influences the perception of vibration for low frequencies up to an excitation of 2 s. Therefore the stimulus duration of the signals are 2 s [Bellmann, 1972].

✓ In the examination of Griffin and Morioka, the difference thresholds were determined with vertical sinusoidal vibration in four conditions: two vibration frequencies (5 and 20 Hz) each presented at two different vibration magnitudes 0.1 and 0.5 ms^{-2} r.m.s. . Subjectes were exposed to a number of trials (about 35 to 60 trials) per threshold determination. A trial consisted of a 4–sec reference stimulus, followed by a 1–sec pause, followed by a 4–sec test stimulus. The order of the reference and the test stimuli was randomized. The magnitude of the reference stimulus was constant at either 0.1 or 0.5 ms^{-2}. The test stimulus was presented at a greater magnitude than the reference stimulus with 0.25 dB (2.9%) increment steps [Morioka and Griffin, 2000].

✓ In the analysis of Mansfield and Griffin, difference thresholds were detected for 20 subjects exposed to four stimuli. Vertical acceleration were produced at the seat guide in a small car on two different surfaces. Subjects were exposed to the following stimulus: 10 s period of a reference motion; 2 s pause; 10 s period of a test motion. The reference and test motions had the same waveform but different magnitude. The magnitude of the test motion was always a little greater than that of the reference motion. The order of presentation of reference and test motions was attributed randomly. Three of the stimuli were provoked by the waveform generated by the motion of car on a tarmac surface. This waveform was reproduced using three different magnitudes of vibration at the seat: 0.2, 0.4 and 0.8 $\mathrm{m\cdot s^{-2}}$ r.m.s. (weighted).

Investigation	Trial
[Bellmann, 1972]	2 s test motion
[Morioka and Griffin, 2000]	4 s reference stimulus, 1 s pause, 4 s test stimulus
[Mansfield and Griffin, 2000]	10 s reference motion, 2 s pause, 10 s test motion

Table 1: Experimental procedures

The other stimulus was provoked by the motion of the car on a pave surface. It obtains a different waveform that was replicated at a magnitude of 0.4 ms^{-2} r.m.s. (weighted) [Mansfield and Griffin, 2000].

2.4 Goodness of Fit

The inferential process proposes a distribution function of the representative process, derived from the sample of observed data. The optimization provides the values of the optimal parameters of the distribution function that approximate the sample of observed data. The inferential process evaluates the uncertainty associated with the distribution function. The results should be interpreted in probabilistic terms for their transferability/generalizability and compared to other conditions survey.

The correctness of the fit has been checked by applying the Pearson correlation coefficient (PCC). The PCC is a measure of the linear correlation (dependence) between experimental data x and results obtained by statistical distribution function y. The PCC test is widely used in goodness–of–fit assessment of statistical distribution function. The PCC has the following form:

$$r = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})}\sqrt{\sum_{i=1}^{N}(y_i - \bar{y})}}, \tag{4}$$

where

- N denotes the number of datapoints;
- $\bar{x}$ is the mean value of experimental data;
- $\bar{y}$ is the mean value of data obtained by using statistical distribution function.

This test is used to measure the strength of a linear association between two variables, where the value $r = 1$ means a perfect positive correlation and the value $r = -1$ means a perfect negative correlation.

3 Results

The comparison illustrated in Fig.2 shows a good convergence between the trend of the average values and statistical distribution functions.

Distribution Function	Logistic	Weibull
Degree of freedom	9	9
p–value	$1.23 \cdot 10^{-8}$	$1.76 \cdot 10^{-7}$
95% confidence interval	0.95–0.99	0.92–0.99
Pearson correlation coefficient	0.99	0.98

Table 2: Pearson's product-moment correlation

The Pearson correlation coefficient provides a probabilistic judgment comparing to the null hypothesis H_0 and the alternative hypothesis H_1, depending on the observed sample. The statistical hypothesis specifies the probability distribution of the random variable. The null hypothesis H_0 represents the status quo respect to the experiment samples. We assume that the null hypothesis H_0 can be represented by the logistic function, because the experimental process consists of a succession of tests with equiprobable and independent events.

The alternative hypothesis H_1 is complementary to H_0. The alternative hypothesis H_1 is represented by the Weibull function, because the order of the observations is randomized. The two hypotheses H_0 and H_1 are not equivalent. The test is never conclusive about H_1. The test relates to the decision to reject or not reject the null hypothesis H_0. The not to refuse H_0 does not mean accepting H_0. If H_0 is not refused, it isn't a support of H_0 (Table 2).

With reference to the numeric values of p–value and the Pearson correlation coefficient we can not refuse H_0. In this study we appreciate p–value and Pearson correlation coefficient by using the *R Project* for statistical computing. The optimal parameters are the following values: location= 0.49 and scale= 0.17 for the Logistic distribution; scale= 0.55 and shape= 2.5 for the Weibull distribution.

4 Discussion

The method of constant stimuli examines the subject's responses. The method of the stimuli provides a direct measure of the proportion of correct answers according to the level of the stimulus. The parameters are the number of presentations in each set of tests, the number of tests. Narrow intensity intervals, for example 1 dB, may not include the threshold level. Intervals with intensity equal to 6 dB could be higher than the threshold level. The method allows to interpret the stimuli response bias. The random testing prevents the risk of repetitive responses. The perceptual process experimental emphasizes low thresholds, involving high sensitivity, and high thresholds, involving low sensitivity. Although the differences in sensitivity can be enhanced by the procedures adopted in the experimental tests.

The examination of the standard deviation and of the variance of the probability $p\,(x)$ of the experimental investigations reveal that standard deviation and of the variance increase in the range 80–84 dB (Fig.3). It means that there is an interval of uncertainty. The range 80–84 dB represents the field where subjects are neither certain that they can feel vibration, nor certain that they cannot feel vibration. The range of uncertainty of a subject, that reports whether he can feel a vibration, depends upon his attitude. The sensitivities to vibration and the range of uncertainty represent two important parts of character or of nature of a subject. For example, we compare the sensitivities to vibration of two subjects. The first subject can be inclined to report that a vibration is present even if he is uncertain. The second subject indicates that he can feel the vibration in the case that he is sure that it is present. The 50% detection is used as the definition of the absolute perception threshold. The first subject reports a lower absolute perception threshold to vibration than the second, even though the subjects have the same sensitivity to vibrations.

Experimental investigations confirm that there is no single vibration magnitude above which a subject will always detect a vibration and below which a subject will not detect a vibration. There is an *interval of uncertainty*. Above such interval of uncertainty the probability that a subject will detect a stimulus increases if the stimulus intensity increases. The subjects have the task of detecting the vibration above his background noise (physiological, acoustic, etc.). The inclination to feel a vibration depends on the subject. These considerations suggest that a statistical distribution function would provide a signal detection theory to determine vibration perception thresholds [Parsons and Griffin, 1988].

5 Conclusion

The psychometric function describes the response behavior of an individual (subject) or group of individuals (mean of subjects) depending on the force of a presented stimulus in a psychophysical experiment. The psychometric function describes the percentage of perceived stimuli that can be characterized by logistic distribution or Weibull distribution. An essential component of the fitting procedure is an assessment of goodness of fit. Goodness of fit is necessary in order to confirm that our estimates of thresholds and slopes, and their variability, are obtained by using a probable model for the data.

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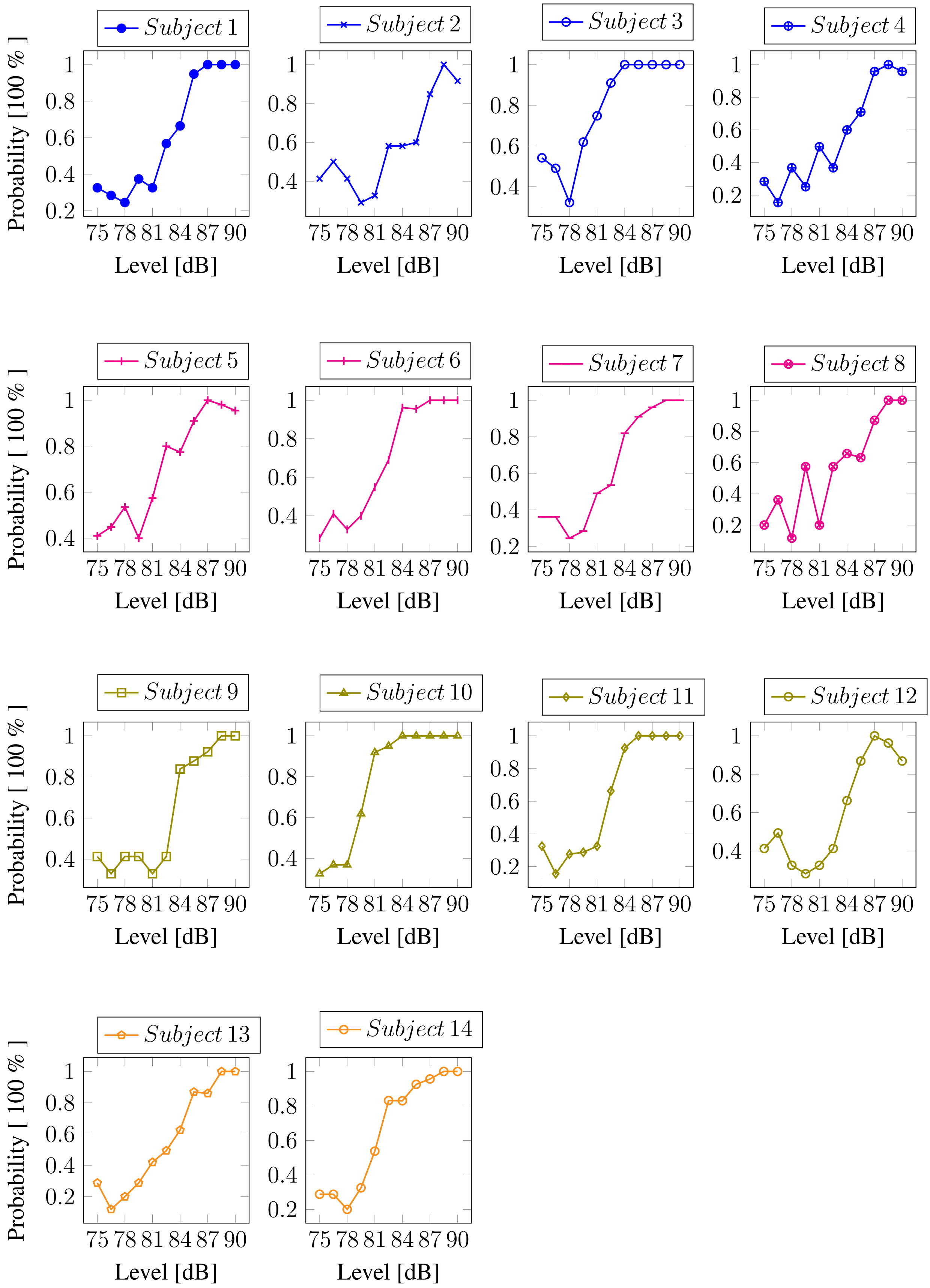

Figure 1: Measured psycometric functions for 14 subjects

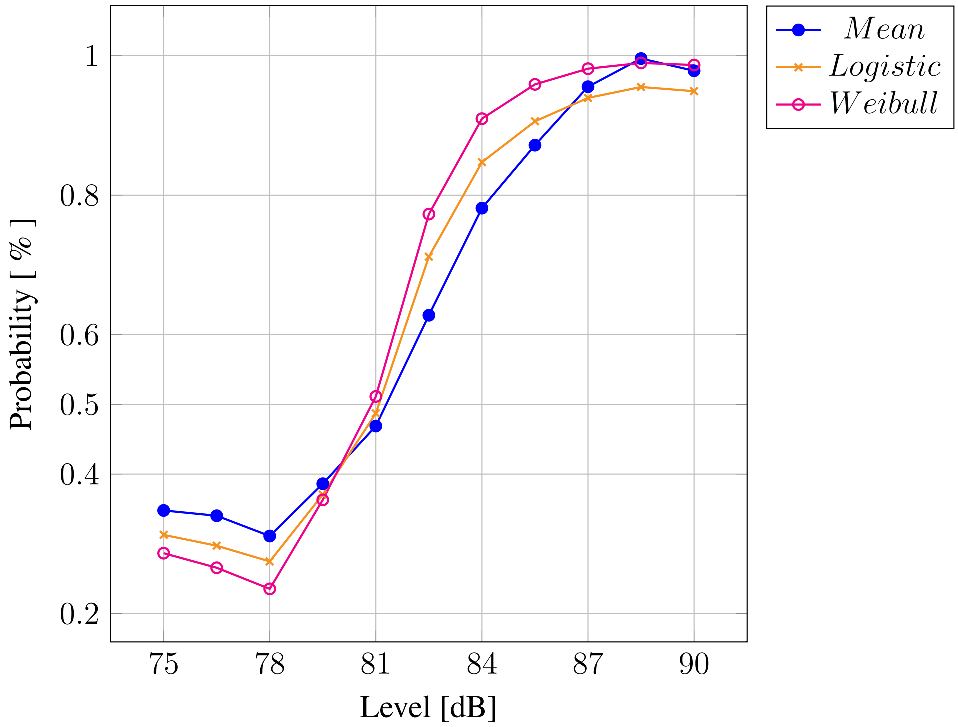

Figure 2: A comparison of statistical distribution functions

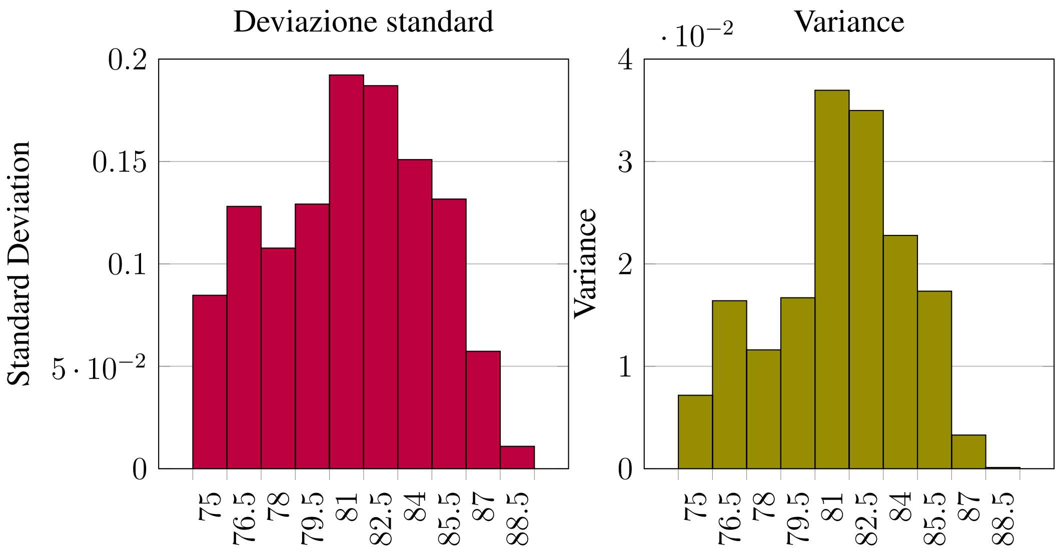

Figure 3: The standard deviation and the variance of the values in $p(x)$